The results obtained at small gas velocities are in qualitative agreement with the existing data for single-phase flow [6]. In a two-phase film flow, the gas component has only slight influence on the temperature distribution in the film, but, by decreasing the thickness of the film, increases the temperature gradient within it.

NOTATION

r, x, radial and axial cylindrical coordinates; H, length of the section of tube considered; R, tube radius; $y = R - r$; δ, film thickness; T, temperature; w, velocity; a, thermal diffusivity; $Re_1 = 4\bar{w}\delta\nu_1^{-1}$; $Re_2 = 2\bar{w}_2 R\nu_2^{-1}$; $a^* = a\nu^{-1}$; $L = HR^{-1}$; $\eta = xR^{-1}$; $h = \delta R^{-1}$; $\varepsilon = yR^{-1}$; $u = w\bar{w}^{-1}$. Indices: l = 1, liquid; l = 2, gas; c, δ, 0, L, R, values at the tube wall, phase interface, tube inlet, tube outlet, and flow axis; a bar over a symbol denotes the mean value.

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APPLICATION OF A CONSERVATIVE DIFFERENCE EQUATION TO DETERMINE NONSTATIONARY HEAT FLUXES

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The possibility of practical utilization of a conservative difference equation of heat conduction, obtained by an integrointerpolation method, for the automatic determination of nonstationary fluxes is analyzed.

The continuous automatic determination of the heat flux is an important problem in the study of nonstationary heat transfer, particularly for the determination of the thermophysical properties (TPP) of substances.

Sufficiently complex algorithms that can be realized only by using digital computer facilities are utilized in existing high-accuracy methods of determining the heat flux density described in [1, 2]. However, utilization of analog apparatus for this purpose is more logical from the viewpoint of fast-response and the simplicity of technical realization. Such an approach to the determination of the heat flux density, based on the solution of the inverse heat conduction problem (IHCP) by using analog facilities was apparently proposed first in [3], but it is impossible to acknowledge the method mentioned as correct.

The simplest method of determining the heat flux, which permits its measurement in analog form during experiment, is based on the following interpolation of the Fourier equation [4]:

$$q(0, \tau) = \frac{\lambda t(x, \tau)|_{x=0}^{x=L}}{L}. \tag{1}$$

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However, this formula results in inadmissibly large errors in investigations associated with the determination of rapidly changing fluxes.

A method is examined in this paper, for the determination of nonstationary heat flux densities at the boundary sections of a plate by means of results of temperature measurements at internal points and on the boundaries, which is free, to a certain extent, of the listed disadvantages. The problem of determining the boundary fluxes for it can be formulated as follows.

There is a homogeneous plate of thickness L with known heat conduction λ and bulk specific heat C in which the heat propagation from a source outside the plate is subjected to a linear one-dimensional heat-conduction equation

$$C \frac{\partial}{\partial \tau} t(x, \tau) = \lambda \frac{\partial^2}{\partial x^2} t(x, \tau), \quad x \in [0, L]. \tag{2}$$

There is information about the temperature differences measured at the plate boundaries

$$t(x, \tau)\Big|_{x=L}^{x=0} = \Delta T(\tau), \tag{3}$$

and the temperatures measured at m internal points

$$t(x_k, \tau) = T_k(\tau), \quad 0 \leqslant x_k \leqslant L \quad (k = 0, 1, \ldots, m-1). \tag{4}$$

It is required to determine the heat flux density passing through the plate boundary

$$q(0, \tau) = -\lambda \frac{\partial}{\partial x} t(0, \tau), \tag{5}$$

$$q(L, \tau) = -\lambda \frac{\partial}{\partial x} t(L, \tau). \tag{6}$$

To solve this problem we use a conservative difference heat-conduction equation obtained by an integrointerpolation method [5]. Let us show the procedure for obtaining such an equation. We apply the operator

$$I(f) = \frac{1}{L} \int_0^L dx \int_0^x f dx \tag{7}$$

to both sides of (2). Substituting (5) in the expression obtained, we arrive at the following equation

$$q(0, \tau) = \frac{\lambda}{L} t(x, \tau)\Big|_{x=L}^{x=0} + \frac{C}{L} \frac{\partial}{\partial \tau} \int_0^L dx \int_0^x t(x, \tau) dx. \tag{8}$$

Let us replace the integrand t(x, τ) in (8) by a Lagrange interpolation polynomial [6]

$$P_{m-1}(x, \tau) = \sum_{k=0}^{m-1} Q_{m-1}^{(k)}(x) t(x_k, \tau), \tag{9}$$

where $Q_{m-1}^{(k)}(x)$ is a polynomial of degree m − 1 determined by the equalities

$$Q_{m-1}^{(k)}(x) = \frac{(x - x_0)(x - x_1)\ldots(x - x_{k-1})(x - x_{k+1})\ldots(x - x_{m-1})}{(x_k - x_0)(x_k - x_1)\ldots(x_k - x_{k-1})(x_k - x_{k+1})\ldots(x_k - x_{m-1})}. \tag{10}$$

We consequently obtain a conservative difference heat-conduction equation from which the heat flux density q(0, τ) entering through the plate boundary can be determined:

$$q(0, \tau) = \frac{\lambda}{L} t(x, \tau)\Big|_{x=L}^{x=0} + \frac{C}{L} \frac{\partial}{\partial \tau} \sum_0^{m-1} p_k t(x_k, \tau). \tag{11}$$

It is easy to show that an analogous difference equation can be obtained to determine the heat flux density q(L, τ) emerging through the plate boundary

$$q(L, \tau) = \frac{\lambda}{L} t(x, \tau)\Big|_{x=L}^{x=0} - \frac{C}{L} \frac{\partial}{\partial \tau} \sum_0^{m-1} p_k^* t(L - x_k, \tau). \tag{12}$$

We substitute (3) and (4) into the difference equations (11) and (12) and obtain computational formulas to find the boundary heat-flux densities

TABLE 1. Time Dependence of the Methodological $\sigma_m(\tau)$ and Summary $\sigma_q(\tau)$ Errors in Determining the Heat Flux in a Section with the Coordinate x = 0 and x = L = 10^{-3} m

τ, sec	$q(x, \tau)$, W/m²·K		$\sigma_M(\tau)$, W/m²·K				$\sigma_q(\tau)$, W/m²·K			
			x=0		x=L		x=0		x=L	
	x=0	x=L	1	2	1	2	1	2	1	2
1	451,36	94,6	2,9	156	7,7	201	9	156	13	201
2	319,2	146,1	1,5	66,8	0,1	106	4,6	67	5,8	106
4	225,7	152,7	0,3	26,3	0,1	46,7	2,4	26,4	3,1	47
9	150,5	126,5	0,1	8,2	0,05	15,7	1,5	8,4	1,6	16
25	90,3	84,8	0,001	1,85	0,004	3,6	0,9	1,9	0,9	3,8

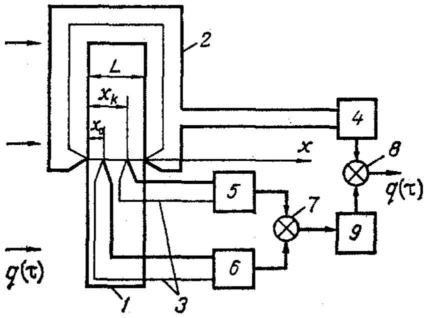

Fig. 1. Apparatus to measure nonstationary heat fluxes.

$$q(0,\ \tau)=\frac{\lambda}{L}\Delta T(\tau)+\frac{C}{L}\frac{\partial}{\partial\tau}\sum_{0}^{m-1}p_kT_k(\tau), \tag{13}$$

$$q(L,\ \tau)=\frac{\lambda}{L}\Delta T(\tau)-\frac{C}{L}\frac{\partial}{\partial\tau}\sum_{0}^{m-1}p_k^*T_k(\tau). \tag{14}$$

As is seen from (13) and (14), the desired heat flux is a linear function in the temperature difference $\Delta T(\tau)$ and the rate of change of the parameter $\sum_0^{m-1}p_kT_k(\tau)$ $(\sum_0^{m-1}p_k^*T_k(\tau))$, which assures a sufficiently simple procedure for its determination.

Let us examine the error in determining the heat flux density be means of the proposed computational formula (13) (the error in (14) is expressed analogously). Let us extract particular errors in measuring the temperature difference $\Delta T(\tau)$ and the rate of change in time of the integral parameter $\int_0^L dx\int_0^x t(x,\ \tau)dx$. We assume that these errors are uncorrelated with respect to each other. Then the total error can be determined from the formula

$$\sigma_q=\sqrt{\left(\frac{\partial q}{\partial y_1}\sigma_{y_1}\right)^2+\left(\frac{\partial q}{\partial y_2}\sigma_{y_2}\right)^2}=\frac{C}{L}\sqrt{(a\sigma_{y_1})^2+\sigma_{y_2}^2}. \tag{15}$$

The second term in the total error, namely, σ_{y_2}, is of interest in this expression. Because the operator $\partial/\partial\tau$ is not bounded, this error, and therefore the total error σ_q in determining the heat flux, can also be infinite.

To eliminate this phenomenon, we use one of the natural regularization modes, which is the approximate calculation of the rate of change of $\sum_0^{m-1}p_kT_k(\tau)$:

$$\frac{d\sum_0^{m-1}p_kT_k(\tau)}{d\tau}\simeq\frac{\sum_0^{m-1}p_kT_k(\tau+\Delta\tau)-\sum_0^{m-1}p_kT_k(\tau)}{\Delta\tau} \tag{16}$$

with preliminary agreement between the computational differentiation spacing $\Delta\tau$ and the error of the input data [1]. In this case the error in determining the rate of change of the integral parameter $\int_0^L dx\int_0^x t(x,\ \tau)dx$ can be found from the following expression:

$$\sigma_{y_2}=\sqrt{\sigma_0^2+2\left(\sum_0^{m-1}p_k\sigma_k/\Delta\tau\right)^2+\varepsilon^2}. \tag{17}$$

A modification of this method of eliminating the instability is the approximate determination of the derivative with respect to the time in analog form, for example, by using apparatus with the transfer function [7]

$$W(p)=\frac{Kp}{Tp+1}. \tag{18}$$

Selecting the coefficients K and T in an appropriate manner, stable operation of this differentiator and sufficiently high differentiation accuracy can be achieved. According to [8], analog differentiating apparatus can assure obtaining results with error at a 1% level. Then the error of measuring the rate of change of the integral parameter $\int_0^L dx\int_0^x t(x,\tau)dx$ can be written in general form as

$$\sigma_{y_2}=\sqrt{\sigma_0^2+\sigma_d^2}. \tag{19}$$

Let us go over to an estimate of the methodological error σ_0 in determining the rate of change of the integral parameter $\int_0^L dx\int_0^x t(x,\tau)dx$. We represent it in the form

$$\sigma_0(\tau)=\frac{\partial\Delta(\tau)}{\partial\tau}, \tag{20}$$

where

$$\Delta(\tau)=\int_0^L dx\int_0^x t(x,\tau)\,dx-\sum_0^{m-1}p_kT_k(\tau). \tag{21}$$

We will consider the approximate formula

$$\int_0^L dx\int_0^x t(x,\tau)\,dx\simeq\sum_0^{m-1}p_kT_k(\tau) \tag{22}$$

to be exact for a zero power polynomial

$$P_0(x,\tau)=a_0=t(x,\tau)|_{x=0} \tag{23}$$

and the rate of change of the heat flux density passing through the plate during the experiment not to exceed a certain constant M:

$$\left|\frac{\partial}{\partial\tau}q(x,\tau)\right|\leqslant M;\quad x\in[0,L];\quad \tau\in[0,\tau_1]. \tag{24}$$

We expand the function $t(x,\tau)$ in powers by the Taylor formula

$$t(x,\tau)=t(x,\tau)|_{x=0}+\int_0^L K(x-z)\frac{\partial}{\partial x}t(z,\tau)\,dz, \tag{25}$$

where

$$K(x-z)=\begin{cases}1 & x-z\geqslant 0,\\ 0 & x-z<0.\end{cases} \tag{26}$$

We substitute (25) into (21) and taking account of (26), we have

$$\Delta(\tau)=\int_0^L dx\int_0^x\int_0^L K(x-z)\frac{\partial}{\partial x}t(z,\tau)\,dzdx-\sum_0^{m-1}p_k\int_0^L K(x_k-z)\times$$
$$\times\frac{\partial}{\partial x}t(z,\tau)\,dz=\int_0^L\left[\int_z^L dx\int_z^x dx-\sum_0^{m-1}p_kK(x_k-z)\frac{\partial}{\partial x}t(z,\tau)\right]dz= \tag{27}$$
$$=\int_0^L\left[\frac{(L-z)^2}{2}-\sum_0^{m-1}p_kK(x_k-z)\frac{\partial}{\partial x}t(z,\tau)\right]dz=\int_0^L F(z)\frac{\partial}{\partial x}t(z,\tau)\,dz.$$

Substituting (27) into (20), we obtain after simple manipulation

$$\sigma_0(\tau)=\frac{1}{\lambda}\int_0^L F(z)\frac{\partial}{\partial\tau}q(z,\tau)\,dz. \tag{28}$$

Taking account of the information about the magnitude of the maximal rate of change in the heat flux density M, i.e., the inequality (24), we turn to an estimate of the methodological error in determining the boundary heat flux density

$$|\sigma_M(\tau)| \leqslant \frac{M}{La} \int_0^L |F(z)|\, dz = \frac{M\alpha}{La}. \tag{29}$$

Therefore, the inequality (29) permit estimation of the methodological error in determining the boundary heat flux density on the basis of information about the maximal rate of change M of the heat flux density in the plate, its thickness L, the thermal diffusivity coefficient a of the material from which the plate is fabricated, and the coordinates x_k of the nodes at which the temperature $T_k(\tau)$ is measured.

It can be shown that the diminution in the methodological error of determining the heat-flux density when using (11) as compared to (1) is

$$\delta(\tau) = \frac{C}{L} \frac{\partial}{\partial \tau} \sum_0^{m-1} p_k T_k(\tau). \tag{30}$$

To illustrate the accuracy of the method proposed, the methodological σ_m and the summary σ_q errors in determining the flux density passing through a section bounding the interval [0, L] of a semiinfinite body with heat conduction $\lambda = 0.32$ W/m·K and volume specific heat $C = 2 \cdot 10^3$ kJ/m^3·K in a known temperature field

$$t(x, \tau) = 1 - \operatorname{erf}\left(\frac{x}{2\sqrt{a\tau}}\right) = \operatorname{erfc}(n) \tag{31}$$

are presented in the table when the following computational formulas are utilized:

Column 1

$$q(0, \tau) = \frac{\lambda}{L} \Delta T(\tau) + \frac{C}{L} \frac{\partial}{\partial \tau} \times \left[\frac{L^2}{6} T(0, \tau) + \frac{L^2}{3} T(0.5L, \tau)\right], \tag{32}$$

$$q(L, \tau) = \frac{\lambda}{L} \Delta T(\tau) - \frac{C}{L} \frac{\partial}{\partial \tau} \times \left[\frac{L^2}{6} T(L, \tau) + \frac{L^2}{3} T(0.5L, \tau)\right], \tag{33}$$

Column 2

$$q(0, \tau) = q(L, \tau) = \frac{\lambda}{L} \Delta T(\tau). \tag{34}$$

The following errors in the input data were taken: 1% in the measurements of the temperature difference $\Delta T(\tau)$, and 5% in the measurements of the rate of change of the parameter $\sum_0^{m-1} p_k T_k(\tau)$. The true values of the heat flux density were determined from the formula

$$q(x, \tau) = -\lambda \frac{\partial}{\partial x} t(x, \tau) = \frac{\lambda \varphi(n)}{2\sqrt{a\tau}}. \tag{35}$$

Calculations were performed by using tables of the Gauss function erf (n) and its derivative $\varphi(n) = d\operatorname{erf}(n)/dn$ with the above-mentioned errors taken into account. The summary error σ_q was determined from (15) and (19) (see Table 1). The high accuracy in determining the heat fluxes can be judged from the example presented.

The apparatus displayed in the figure can be used for automatic measurement of the boundary heat flux density passing through a plate. The apparatus contains a primary transducer, which is the plate 1, from a material with known TPP, a differential thermocouple 2 measuring the temperature difference on the plate boundaries the thermocouple 3 whose working junctions are mounted over the plate thickness at given points x_k, the thermal emf transducers 4, 5, 6, where the transduction factors of 5, 6 are proportional to the weights p_k, the adders 7, 8, the apparatus 9 to measure the rate of change of the input signal. As follows from the description, this apparatus realizes one of the formulas (11) or (12) depending on the boundary at which the heat flux density is being measured.

In conclusion, let us note the main achievements of the methodology of determining the heat-flux density, based on application of a conservative difference heat-conduction equation.

Firstly, the possibility of estimating the error in determining the heat-flux density by means of the analytic expressions (15), (17), (19), which affords a possibility of constructing heat meters with given accuracy. Secondly, simple computational formulas permitting the realization of continuous measurement of nonstationary heat fluxes during experiment by using analog facilities.

NOTATION

$p_k = \int_0^L dx \int_0^x Q_{m-1}^{(k)}(x)dx$; $p_k^* = \int_L^0 dx \int_L^{L-x} Q_{m-1}^{(k)}(L-x)dx$, weighting factors; $y_1 = \Delta T(\tau)$; $y_2 = \frac{\partial}{\partial \tau} \int_0^L dx \int_0^x t(x, \tau) dx$; σ_{y_1}, error in measuring $\Delta T(\tau)$; σ_{y_2}, error in determining y_2; a, thermal diffusivity coefficient; σ_0, methodological error in determining y_2; σ_k, error in measuring $T_k(\tau)$; ε, error in determining the derivative due to piecewise-linear interpolation; and σ_d, error in approximate analog differentiation.

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DERIVING THE THERMAL CONTACT RESISTANCE FROM THE SOLUTION OF THE INVERSE HEAT-CONDUCTION PROBLEM

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The construction of an iterative computational algorithm is considered, and results of mathematical modeling of the solution of the coefficient inverse problem of heat conduction by deriving the dependence of the thermal contact resistance on the temperature are given.

Consider the process of heat conduction in a two-layer infinite plate with known thermophysical characteristics of the layers and specified initial and boundary conditions of the first kind.

In real situations, there is contact heat transfer between the layers at the boundary. This means that, in numerical modeling of the heat-conduction process in the system, the energy-matching relations at the boundary between the layers must be considered, taking account of contact thermal resistance [1]. It is assumed that the heat conduction in each layer is described by a homogeneous heat-conduction equation. Then the mathematical formulation of the problem of heat conduction in a two-layer plate takes the following form for the given case

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